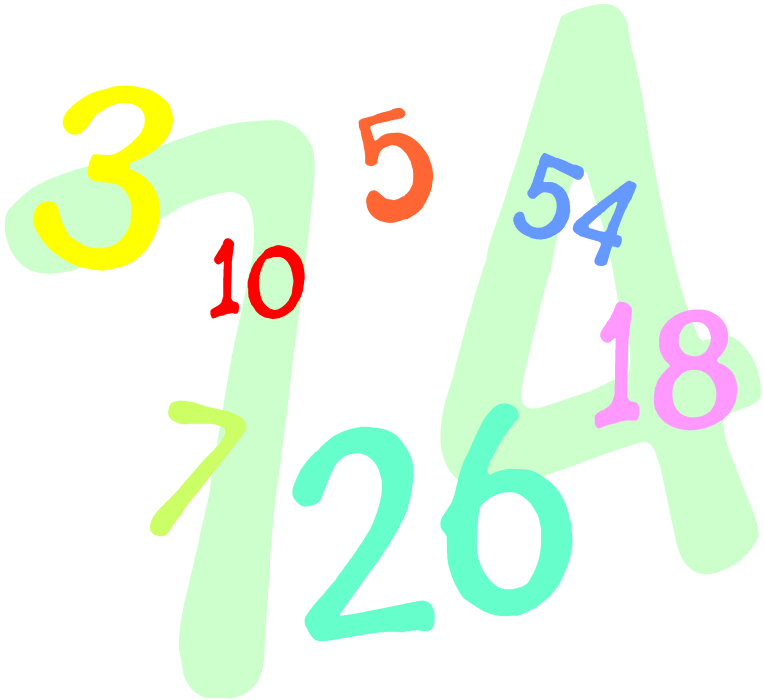
OXFORDSHIRE COUNTY COUNCIL

The Primary Support Team

Continuing Professional Development Standard
National Centre for Excellence in the Teaching of Mathematics
Professional Development Accredited Lead
National Centre for Excellence in the Teaching of Mathematics

Calculation Policy

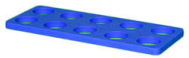
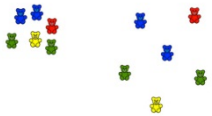


Introduction:

Children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both **conceptual understanding** and **fluency** in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use models and images to support their mental and written methods of calculation. As children's mental methods are strengthened and refined they begin to work more efficiently, which will support them with using succinct written calculation strategies as they are developed.

From Early Years to Year 1:

There are fundamental skills that it is important for children to develop an early understanding of as building blocks to future learning in maths, including that linked to calculation. A selection of the skills include:

- Ordinality – 'the ordering of numbers in relation to one another' – e.g. (1, 2, 3, 4, 5…)
- Cardinality – 'understanding the value of different numbers' – e.g. (7 =  17 =  +  12 = 
- Equality – 'seven is the same total as four add three' – e.g.  = 
- Subitising – 'instantly recognizing the number of objects in a small group, without counting them' – e.g.  → five
- Conservation of number – 'recognising that a value of objects are the same, even if they are laid out differently' – e.g. 
- One-to-one correspondence – e.g. 
- Counting on and back from any number – e.g. 'five add three more totals eight'  'ten take away three totals seven' 
- Using apparatus and objects to represent and communicate thinking – e.g.  = 10
- Maths language – using mathematical words verbally in every-day situations – e.g. 'climb up to the top' / 'climb down to the bottom'

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice through progression in relevant practical maths experiences and visual representations.

By the end of Year 6, children will be equipped with efficient mental and written calculation methods, which they use with fluency. Decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. At whatever stage in their learning, and whatever method is being used, children's strategies must still be underpinned by a secure understanding and knowledge of number facts that can be recalled fluently.

The overall aims are that when children leave primary school they:

- Are able to recall number facts with fluency, having developed conceptual understanding through being able to visualise key ideas – such as those related to place value - through experience with practical equipment and visual representations;
- Make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- Have an efficient, reliable, written method of calculation for each number operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;
- Are able to make connections between all four number operations, understanding how they relate to one another, as well as how the rules and laws of arithmetic can be applied.

Disclaimer:

This draft calculation policy has been structured by members of the Oxfordshire Primary Support Team, taking into account statutory requirements as detailed in the new 2013 National Curriculum for Maths. It has been set out to highlight general progression in calculation, which will allow pupils to develop conceptual understanding through continued use of practical equipment and visual representations.

The policy has a correlation to year-by-year expectations set out in the national curriculum programmes of study; with some additional steps. However, schools are encouraged to personalise this policy, taking into account that statutory elements will need to be maintained.

Oxfordshire Primary Support Team's **'progression charts'** and **'on the boil'** documents illustrate year-by-year requirements and should inform this policy. 'On the boil' documents give a much broader range of year specific mental recall ideas and expands on the rapid recall section of this policy.

If mathematical structures such as the bar model are to be used, ideally as a whole-school system of learning and teaching, then it is advised that schools engage with continuing professional development opportunities in the first instance. The Oxfordshire Primary Support Team's **'progression in use of the bar model'** document can be used as a reference point by schools in addition to content that can be found on the NCETM's website.

It is important that we all use the same vocabulary when referring to formal written methods – as listed in this document.

In the Autumn term; complete a baseline assessment to check that children have met previous years expectations and use this to inform planning. Don't be afraid to look at the stage before if children need time to consolidate their understanding and use differentiation where needed. Expectation is that children will meet each stage before moving on.

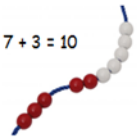
It is not intended for children to quickly move up the stages; children need to be competent and confident in each stage and be able to show they can tackle a range of activities and questions to show a deeper understanding.

Cross reference this policy against the National Curriculum to ensure that you are teaching all the curriculum.

Mental Calculation Strategies for Addition and Subtraction

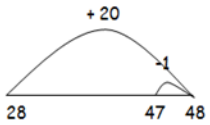
Number Bonds

7 + 3 = 10



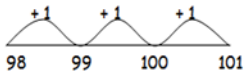
Adjusting

28 + 19 = 47

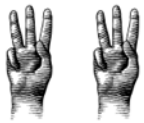


Finding the Difference

101 - 98 = 3

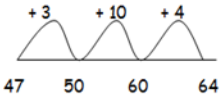


Doubles



Bridging

47 + 17 = 64



Near Doubles

Partitioning

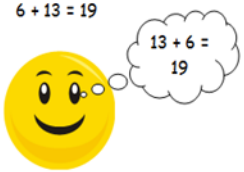
44 + 34 = 78

70 + 8 = 78

Reordering

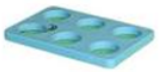
e.g. put big number in head when counting on

6 + 13 = 19



Mental Calculation Strategies for Multiplication and Division

Knowing multiplication and division facts to 12 X 12

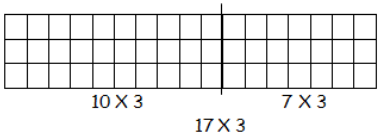


Multiplying and dividing by multiples of 10

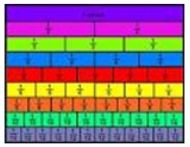
thousands	hundreds	tens	ones

Doubling and halving

Multiplying and dividing by single-digit numbers and multiplying by two-digit numbers

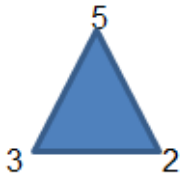
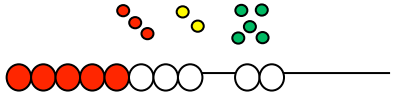
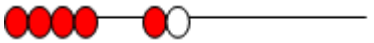


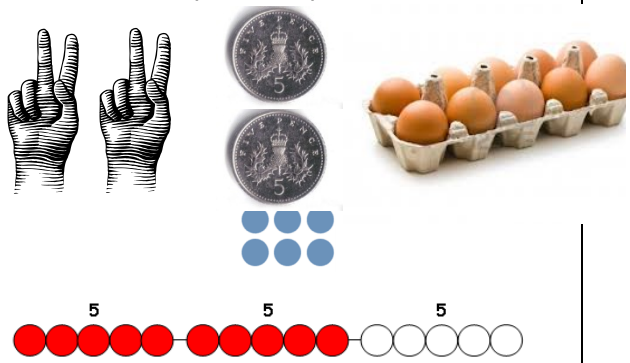
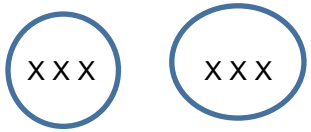
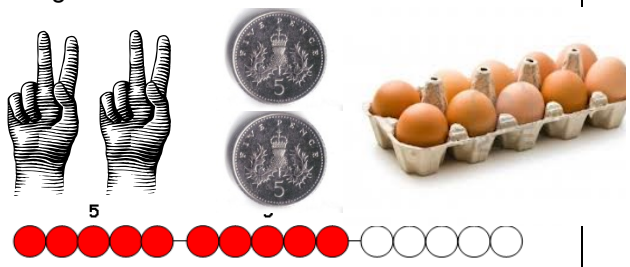
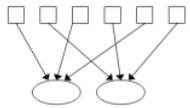
Finding fractions, decimals and percentages



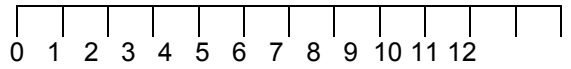
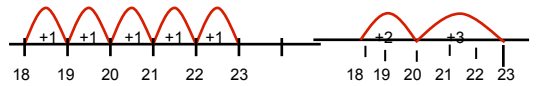
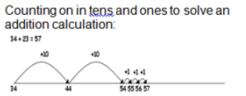
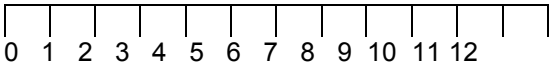
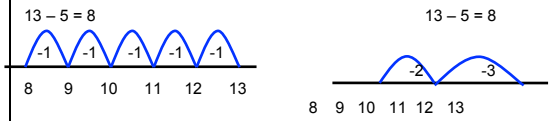
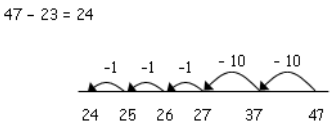
Written Calculation Methods			
Addition	Subtraction	Multiplication	Division
Number Sentences Number lines Expanded column method Column (compact) method	Number Sentences Number lines Expanded column method Column method	Arrays Grid Long Multiplication Short Multiplication	Grouping and sharing Arrays Chunking Long division Short division

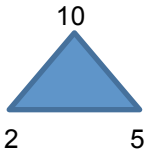
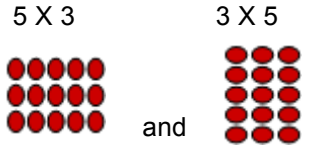
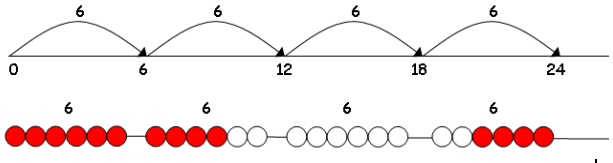
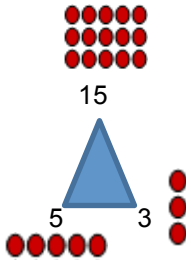
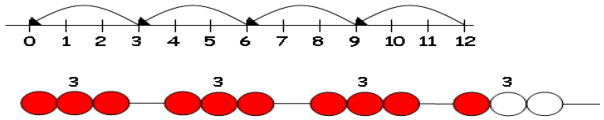
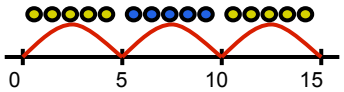
Stage 1

	Counting	Mental maths strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Addition	Count in ones to and across 100 **forwards** and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten.	Pupils use apparatus to explore addition as the inverse of subtraction.  	Derive then, recall all pairs of numbers totalling up to 10. (story of 5,6,7,8,9,10) Use structured apparatus – i.e. Numicon, tens frames, abaci, etc.	**Combining two groups:** • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment. • Model use of number tracks to count on or line up counters/objects along the number track. This is a precursor to use of a fully numbered number-line. Record in number sentences	 3 + 2 = 5  'eight add two more makes ten'  'one more than four is five' Children draw around numicon
Subtraction	Count in ones to and across 100, forwards and **backwards** starting from 0, 1 and other numbers. Count back in tens and begin to subtract 10 from any 2-digit number. Count in multiples of two, five and ten.	 4 add 1 is 5 5 subtract 4 leaves 1	Derive, then recall of subtraction facts for numbers up to 10. (story of 5,6,7,8,9,10) Use structured apparatus, i.e. Numicon, tens frames, abaci etc.	**Subtraction as taking away from a group:** • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment. • Model use of number tracks to count back or remove counters/objects from the number track or set. This is a precursor to use of a fully numbered number-line Record in number sentences	  5 - 2 = 3  'six take away two leaves four'  'one less than six is five'

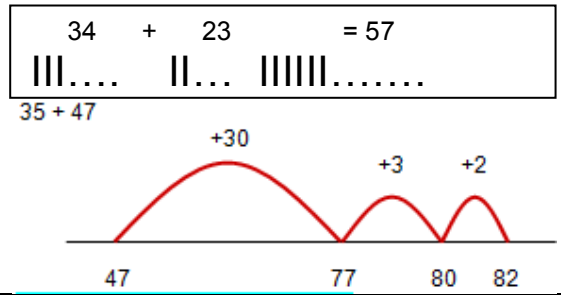
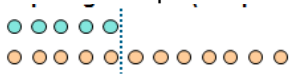
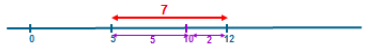
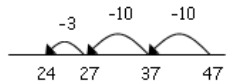
Multiplication	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to ten	**Developing early conceptual understanding of multiplication:**	Use objects, pictorial representations and arrays to show the concept of multiplication: 
Division	Count forwards and backwards in 2s, 5s and 10s	Halving up to twelve and then twenty whilst using related models and images. Find half of even numbers by sharing equally into two groups	Derive/recall half of even numbers to twenty Recall odd and even numbers to 10 in reference to structured apparatus. 	**Developing early conceptual understanding of division as grouping and sharing:** Sharing into groups 6÷2=3 	Use objects, pictorial representations and arrays to show the concept of division as grouping and sharing.  “Two children share six pencils between them”  “Six children are asked to get into three equal groups” 

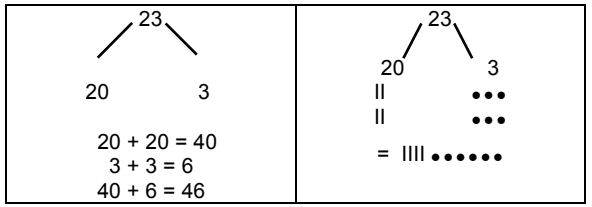
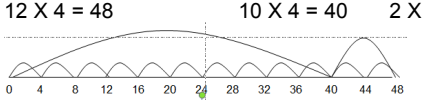
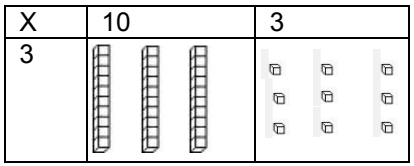
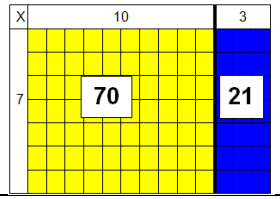
Stage 2

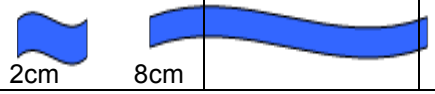
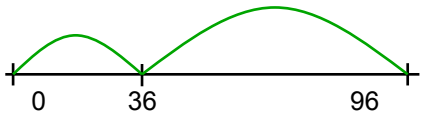
	Counting	Mental maths strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Addition	Continue practicing above skills. Count in steps of 2, 3 and 5 forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy bank as well as a number square.	Reorder numbers when adding, i.e. start with largest number, find bonds, etc. Add doubles and derive near doubles. Round numbers to the nearest 10. Add 3, one-digit numbers, spotting doubles, number bonds	Recall addition facts for all numbers to 20.	**Counting on from any number:** • Children begin to use numbered lines to support their own calculations, initially counting in ones before beginning to work more efficiently. **Counting on from the largest number:** • Children reorder calculations to start with the largest number.	Number line with all numbers labelled  18 + 5 …to…    Use of questions such as: 'How might I rearrange these to find the total?'
Subtraction	Continue practicing above skills. Count in steps of 2, 3 and 5, forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy bank, use dienes, numicon, as well as a number square.	Bridging through 2-digit numbers, i.e. 56- 3 = 53-5= (53-3-2) using number lines. Subtract 10 and small multiples of 10 from any 2-digit numbers. Subtract any pair of 2-digit numbers by counting back in steps of 10 & 1. Use knowledge of place value to solve 35-5=	Recall subtraction facts for all numbers to 20.	**Subtracting by counting back and on:** • Children begin to use numbered lines to support their own calculations, initially counting back in ones before beginning to work more efficiently. • On a number line, including - Subtracting 11 by subtracting 10 and then 1 more. • Then move onto using blank number lines **Subtracting by TU-U and TU-TU**	Number line with all numbers labelled  Using blank numbers lines  Counting back in tens and ones to solve a subtraction calculation: 

Multiplication	Continue practicing above skills. Begin to count forwards and backwards in 3s, from zero.	Begin to understand and use inverse number operations:  Stories are used alongside a triad to help children understand links between number operations, e.g. "There are five pencils in two packs, which means that there are ten pencils altogether."	Derive/recall doubles to twenty, then multiples of ten to fifty Recall & use multiplication facts for the 2X, 5X and 10X-tables.	**Understanding multiplication as repeated addition:** • Investigate multiplication as repeated addition, using arrays so that the law of commutativity is understood. • Whilst arrays are also modelled explicitly at this stage, it is important to note that they will continue to be a key model at later stages, alongside more formal methods of calculation.	Arrays:  Number lines: 6 X 4 = 24  So: 'Six taken four times"
Division	Continue practicing above skills.	Begin to understand and use inverse number operations.  Stories are used alongside a triad to help children understand links between number operations, e.g. "15 children are asked to get into three groups and find out that there are five people in each group."	Begin to half numbers to 40 and multiples of 10 to 100 Recall & use division facts derived from 2X, 5X and 10X-tables. Recall odd and even numbers to 10 in reference to structured apparatus.	**Understanding division as repeated subtraction:** • Investigate division as repeated subtraction. • Through teacher modelling, children need to know that division is not commutative.	Number lines and arrays: 12 ÷ 3 = 4  15 ÷ 5 = 3 

Stage 3

	Counting	Mental maths strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Addition	Continue practicing above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on by 10 or 100 from any two digit number. Link to counting stick: counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.	Partitioning by bridging through 10 and multiples of 10 when adding. Adjusting when adding 11 or 9 to a number. Relating inverse number operations – using structured apparatus to explore and understand that subtraction undoes addition.	Connect pairs totalling ten to pairs of multiples of 10 totalling 100. Use 10ps in tens frame. Recall pairs of two-digit numbers with a total of 100, i.e. 32 + ? = 100.	**Add any pair of 2-digit numbers** **Expanded horizontal addition:** • Add numbers using structured apparatus to support understanding of place value. • Make connections between partitioning both numbers using structured apparatus and partition the second number only using a number line.	Add…  …and…   By partitioning and recombining 30+ 40 = 70 5 + 7 = 12 70 + 12 = 82 
Subtraction	Continue practicing above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on and back by 10 or 100 from any two digit number. Link to counting stick counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.	Partitioning by bridging through 10 and multiples of 10 when subtracting. Continue to practice adjusting when subtracting 11 or 9 from a number. Perform place value subtractions without a struggle, e.g. 127-20 =107,	Continue practicing recall of number facts to 20, including associated subtraction facts Connect subtractions from ten to subtractions from multiples of 10 totalling 100.	**Finding the difference:** • Teachers model how to find the difference when two numbers are relatively 'close together.' • Initially children compare two sets before moving on to a number line comparison. • **Pupils are taught to choose whether to count on or back** depending on which is more efficient. **Subtracting TU-U, TU - TU**	Comparing two sets: comparison or difference.  Finding the difference on a number line.  Note: Finding the difference is often the most efficient way of solving a subtraction problem, e.g. 61 – 59 2,003 – 1,997 Using a blank number line and counting back more efficiently: 47 – 23 = 24 

Subtraction		Relating inverse number operations – use structured apparatus to explore and understand that subtraction undoes addition. Subtract 1p, 10p, £1 from amounts of money, finding change from £1,£5, £10			
Multiplication	Counting forwards and backwards in 4s, 8s from zero. Count up and down in tenths.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Multiply by 10 and 100	Recall odd and even numbers to 100 in reference to structured apparatus. Recall and use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables. Begin to double multiples of 5 to 100, then 2-digit numbers of less than 5	**Relate multiplying a 2-digit by 1-digit number using repeated addition and arrays to represent:** Doubling by partitioning 	Children use an empty number line to chunk efficiently: 12 X 4 = 48 10 X 4 = 40 2 X 4 = 8  3 X 13 = 39  7 X 13 = 91 

		Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon  			
Division	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Divide by 10 and 100 to give whole numbers Introduce the structure of scaling: e.g. Find a ribbon that is 4 times shorter than the blue ribbon.  	Recall odd and even numbers to 100 in reference to structured apparatus. Know by heart all the division facts derived from 2X, 3X, 4X, 5X, 8X and 10X tables.	**Dividing a 2-digit by 1-digit number, representing this efficiently on a number line:**	Children use an empty number line to chunk efficiently. 96 ÷ 6 = 16 6 x 6 = 36 10 x 6 = 60 

Stage 4

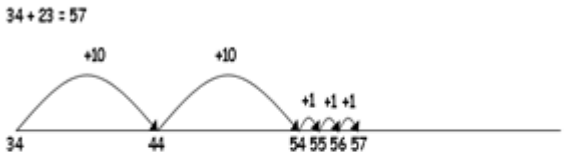
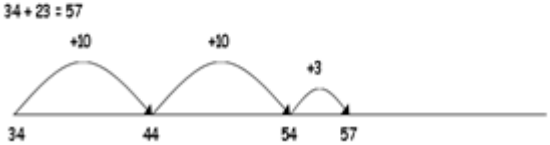
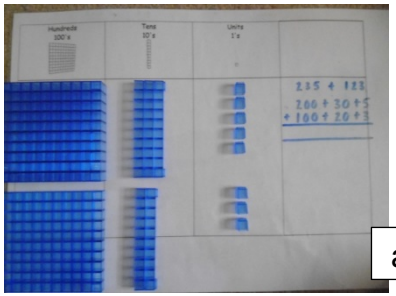
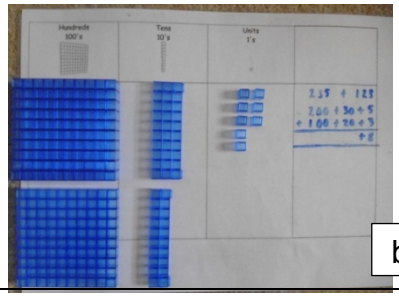
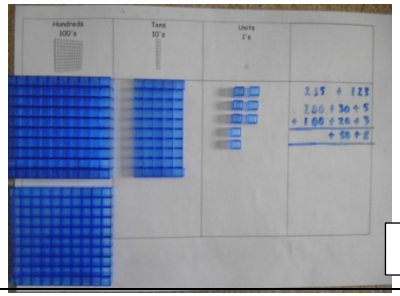
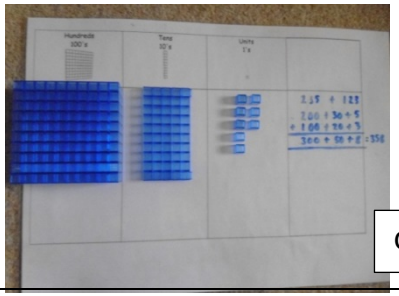
	Counting	Mental maths strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Addition	Continue practicing previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes equipment, counting stick, ITPs.	Compare times in terms of seconds, minutes and hours - Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes. Rounding any number to the nearest 10, 100 or 1000 and numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers. Add pairs of friendly 3-digit numbers, e.g. 320+400 Begin to add amounts of money, using partitioning.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 8 + 3 = 11, I also know 0.8 + 0.3 = 1.1 and 8/100 + 3/100 = 11/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Addition doubles of numbers to 100. Pairs of fractions totalling 1.	**Add pairs of 3 digit numbers** **Expanded horizontal method, leading to column addition:** • Written recording should follow teacher modelling around the size of numbers and place value using a variety of concrete materials, e.g. straws, Numicon, Dienes and place-value cards. • Teachers model how numbers can be partitioned into tens and ones, as well as in different ways, e.g. 20 + 5 10 + 15 • As children move towards using a columnar method, links continue to be made with earlier models and images, including the number line.	It is crucial that empty number lines are kept as well as using more formal written calculation methods. Counting on in tens and ones to solve an addition calculation:  Counting on more efficiently: 

Illustration of how to use Dienes equipment to ensure children have an understanding of place value when using columnar addition.

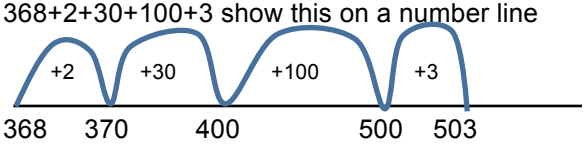
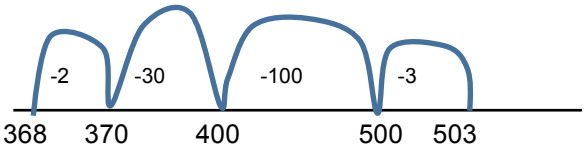
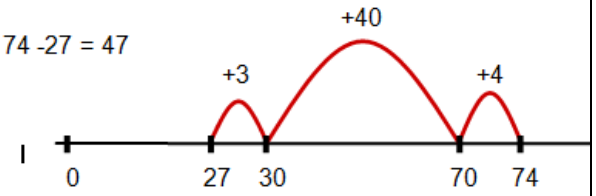

a


b


c

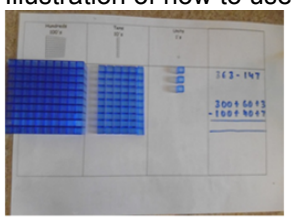
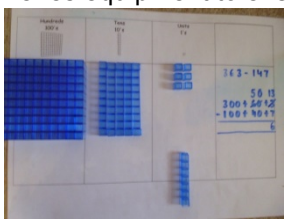
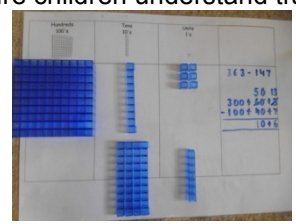
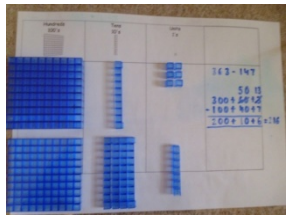

d

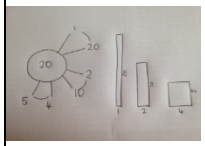
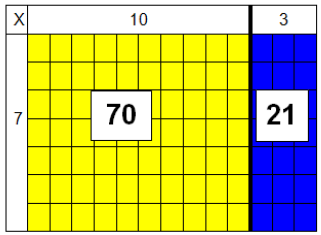
Subtraction

Continue practicing of previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes equipment, counting stick, ITPs.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers. Perform place value subtractions without a struggle, e.g. 4736-706 =4030, Also finding change from £10, £20, £50	As above. Use known facts and place value to derive new ones, i.e. 'If I know 11 - 3 = 8, I also know 1.1 - 0.3 = 0.8 and 8/100 - 3/100 = 5/100.' Sums and differences of pairs of multiples of 10, 100 or 1000.	**Subtracting HTU – U, HTU – TU and HTU and HTU:** Using counting up or back (pupil choose the most efficient way) as an informal written strategy for subtracting pairs of 3 digit numbers e.g. 503-368 by addition is 368+2+30+100+3 show this on a number line  503-368 by subtraction is 503-3-100-30-2=  As children move towards using a column method, links continue to be made with earlier models and images, including the number line.	Use empty number lines to find the difference by bridging through multiples of ten.  Subtract by starting with the first number and partitioning the second, i.e. 74 - 27 74 – 20 = 54 54 – 4 = 50 50 – 3 = 47

363 – 147 = 216
50 13
300 + 60 + 3
100 + 40 + 7
200 + 10 + 6 = 216

Illustration of how to use Dienes equipment to ensure children understand transference of numbers when using column subtraction

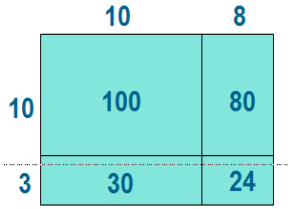
Multiplication	Counting forwards and backwards in 7s, 25s and 1000s from zero. Count up and down in tenths and hundredths.	Derive factor pairs of numbers using models and images, e.g.  Know what happens when a number is multiplied by zero or one. Use reordering to multiply three numbers. Multiply whole numbers and one place decimals by 10, 100, 1000	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 3/2-digit by 1-digit number with arrays towards using long/short multiplication:**	Relate multiplying a 3/2-digit by 1-digit number, now also setting it out as short multiplication.  7 X 13 = 91 7 X 10 = 70 7 X 3 = 21 = 91 At this stage, the **non-statutory** guidance in the national curriculum suggests teaching short multiplication; however, the team feel that an expanded form of calculation (as set out above) is be a better lead into long/short multiplication. Expanded Column (Ladder)
Division	Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.	Derive factor pairs of numbers using models and images. Know what happens when a number is divided by zero or one. Use place value and number facts in mental division i.e. 84 ÷ 4 is half of 42	Recall & use division facts derived from all times-tables up to 12 X 12. Half even numbers to 100 and odd numbers to 20	**Dividing a 3/2-digit by 1-digit number, representing this efficiently on a number line, also in relation to long division:** • At this stage, no remainders are present unless in a practical context.	Children use an empty number line to chunk efficiently. 224 ÷ 8 = 28 8 x 8 = 64 20 x 8 = 160  28 8 224 - 160 (8 X 20) 64 - 64 (8 X 8) 0 …or… 28 8 224 20 X 8 = 160 64 8 X 8 = 64

.

Stage 5

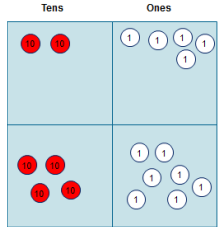
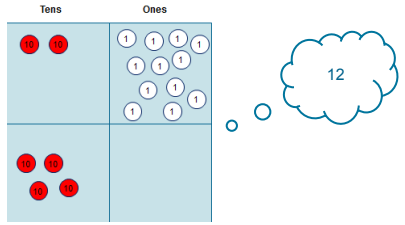
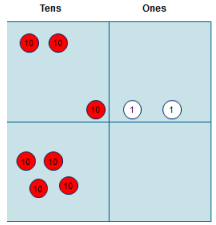
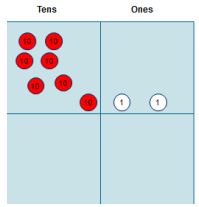
	Counting	Mental maths strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Addition	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.	Use apparatus and knowledge of place value to add decimals, i.e. 3.8 + 2.5 = 5 + 1.3, including on a number line Reorder increasingly complex calculations, i.e. 1.7 + 2.8 + 0.3 = 1.7 + 0.3 + 2.8 Add numbers with 2 significant digits only, using mental calculations, e.g. 3.4 + 4.2 23 000 + 47 000 Compensating – i.e. 405 + 399 → add 400 and then subtract 1.	Continue to practice previous stage and make links between known facts and addition pairs for fractions, percentages and decimals Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7	**Add 4 digit numbers** **Expanded column (vertical) method, leading to column addition:** • Teachers model a column method that records and explains partial mental methods. • There remains an emphasis on the language of calculation, e.g. 'Forty plus seventy equals one-hundred and ten.'… 'Seven add six equals thirteen.' …before recombining numbers. Teachers also model the language of: 'Four tens add seven tens total eleven tens or 110.' • Teachers similarly advance to model the addition of two 3-digit numbers with the expectation that as children's knowledge of place value is secured, they become ready to approach a formal compact method. Include adding amounts of money to decimal places Add like fractions 3/5 + 4/5=7/5=1 2/5 Use complimentary addition to subtract amounts of money and for subtractions where the larger numbers is a near multiple of 1000 or 100, e.g 2002-1865=	Informal column: Adding the tens first: 47 + 76 110 13 123 Adding the ones first: 47 + 76 13 110 123 £3.57 + £4.22

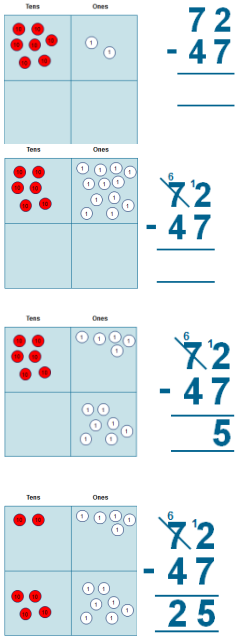
Subtraction	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.	Use apparatus and knowledge of place value to subtract decimals, i.e. 3.8 - 2.5 = 1.3, including money Reorder increasingly complex calculations, i.e. 1.7 – 0.5 – 0.7 = 1.7 – 0.7 – 0.5 Compensating – i.e. 405 - 399 → subtract 400 and then add 1. Subtract numbers with 2 significant digits only, e.g. 6.2-4.5= 72000 - 47000= Subtract 1 or 2 digit near multiples of 10, 100, 1000, 10000, 100 000 from other numbers e.g.82472-30004=	Continue to practice previous stage and make links between known facts and addition pairs for fractions, percentages, decimals and money. Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7	**Continue expanded method, leading on to compact method, up to 5 digits:** • Written recording should follow teacher modelling around the size of numbers and place value using a variety of concrete materials, e.g. straws, Numicon, Dienes and place-value cards. Subtract like fractions 4/5 - 3/5=1/5 Use complimentary addition for subtraction of amounts of money and for subtractions where the larger numbers is a near multiple of 1000 or 100, e.g 2002-1865=	Children should continue to use empty number lines and use more formal written methods when numbers become too big or complex.

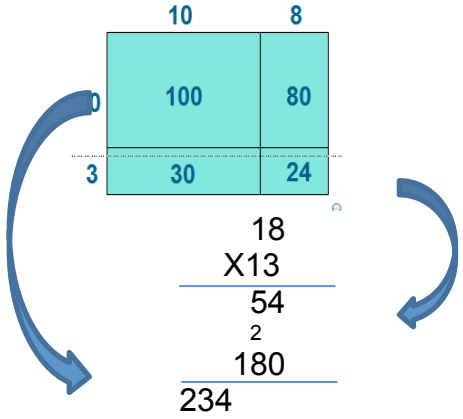
Multiplication	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify and use multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. Multiply near multiples by rounding eg 33x19 as (33x20) -33 Begin to double amounts of money e.g. £35.60 doubled is £71.20 Multiply whole numbers and one and two place decimals by 10, 100,1000,10 000 Partition decimals numbers to multiply by a 1 digit number, e.g. 6.3x7 = 6x7 = 0.3x7 Use doubling and halving as a strategy for mental multiplication e.g. 58 x 5 is half of 58 x10	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 4/3/2-digit by 1/2-digit number with grid to using long multiplication, choosing the most efficient method:**	 18 X13 24 30 80 100 234

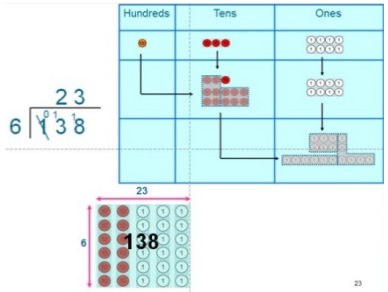
Division	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. Divide multiples of 100 by a 1-digit number using division facts e.g. 32,000 ÷ 8 = 400 Know square numbers and cube numbers Halving amounts of money by partitioning	Recall & use division facts derived from all times-tables up to 12 X 12. Half of even numbers to 200	**Dividing a 4/3/2-digit by 1-digit number, in relation to long division:** • By this stage, there is a statutory requirement that children can use a formal written calculation method, such as long division. • Short division may begin to be taught alongside long division, but still with use of visual representations	As schools have autonomy to decide children's progression in learning between long and short division in Years 5 and 6, the maths team suggest beginning with long division. Remainders should be interpreted in the following ways when long division is used: • as whole numbers • as fractions • through rounding in an appropriate way to the context Long division: 415 ÷ 9 = 46 and 1/9 46 and 1/9 9 ⟌ 415 - 360 (9 X 40) 55 - 54 (9 X 6) 1

Stage 6

	Counting	Mental maths strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Addition	Continue to practice previous skills. Count **forwards** and backwards in simple fractions, decimals and percentages.	Partitioning using near doubles, i.e. 2.5 + 2.6 = 5 + 0.1 Reorder decimals, i.e. 4.7 + 5.6 – 0.7 …as… 4.7 – 0.7 + 5.6 = 4 + 5.6.	Make links between decimals, fractions and percentages. Know by heart number bonds to 100 and use these to derive related facts, e.g. 3.46+0.54 Add negative numbers in a context such as temperature where the numbers make sense, showing on number lines and in jottings as needed	**Columnar addition (formal written method):** • The concept of exchange is introduced through continued use of practical equipment (manipulatives). • Teachers model: 1. "I have two tens and five ones, which need adding to four tens and seven ones." 2. "I add five ones to seven ones, which gives me twelve ones." 3. "I exchange ten of my twelve ones for a ten counter." 4. "I add my three tens and four tens to make seven tens." "Altogether, I have seven tens and two ones." • Teachers similarly advance to model the addition of two 3-digit numbers, e.g. 587 + 475 1062 1 1 Add numbers with up to 5 digit numbers and with decimal numbers to three places. Choose the most efficient method in any given situation Add mixed fractions with different denominators, e.g. 1 ½ + 2 ¾ 1 $^{2}/_{4}$ + 2 ¾ 3 $^{5}/_{4}$ = 1¼ 3 + 1 ¼ = 4 ¼	**Pupils to be encouraged to consider mental strategies first.** Compact/Column method: 25 +47  25 +47 2 1  25 +47 2 1  25 +47 72 1 

Subtraction	Continue to practice previous skills. Count forwards and **backwards** in simple fractions, decimals and percentages.	Use number bonds to 1 and 10, to perform mental subtraction of any pair of 1 place or 2 place decimal numbers using complimentary addition and including money. E.g 10-3.65 as 0.35+6 £50-£34.29 as 71+15 Subtract multiples of powers of 10 and near multiples of the same Subtract negative numbers in a context such as temperature	Make links between decimals, fractions and percentages, drawing on children's knowledge of basic number facts and place value	**Second stage of column method:** • The concept of exchange is introduced through continued use of practical equipment (manipulatives). • Teachers model: 1. "I have seven tens and two ones. I need to subtract four tens and seven ones." 2. "At the moment, I cannot subtract seven ones from two ones, so I need to transfer one ten to become ten ones." 3. "Now I can take away seven ones from twelve ones, so that I have fives ones left. 4. "I can now subtract four tens from six tens, which leaves me with two tens." 5. "I recombine two tens and fives ones to understand that I am left with twenty-five." • Teachers similarly advance to model the subtraction of one 3-digit number from another. Use complimentary addition for subtraction of amounts of money and for subtractions where the larger numbers is a near multiple of 1000 or 10 000, e.g 2002-1865= Subtract mixed numbers and fractions with different denominators	Column Method: 

Multiplication	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and operations. Multiply whole numbers and decimals with up to 3 places decimals by 10, 100,1000 Double decimal numbers with up to 2 places add example	Recall & use multiplication facts for all times-tables up to 12 X 12. In addition, use facts confidently to make larger calculations.	**Use short multiplication to multiply a 1-digit number with up to 4 digits, (including money)** **Use long multiplication to multiply a 2-digit number by a number with up to 4 digits** **Selecting the most efficient method**	

Division	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and different number operations. Divide whole numbers by 10, 100, 1000 to give whole numbers answers or answers with 1,2,3 decimal places Identifying common factors, common multiples and prime numbers Divide 1 and 2 place decimals, including money, by numbers up to and including 10, using place value	Recall & use division facts derived from all times-tables up to 12 X 12. In addition, use facts confidently to make larger calculations. Use doubling and halving up to 2-decimal places as mental division strategies.	**Dividing a 4/3/2-digit by 2/1-digit number, in relation to long and then short division:** • By this stage, there is a statutory requirement that children can use formal written calculation methods, including long and short division. • Use of visual representations – like the ones opposite – remain important. Choose the most efficient method in any given situation	Remainders should be interpreted in the following way when short division is used: • through rounding in an appropriate way to the context Long division: 432 ÷ 15 = 28 4/5 2 8 1 5 | 4 3 2 3 0 0 15×20 1 3 2 1 2 0 15×8 1 2 $\frac{12}{15} = \frac{4}{5}$ Answer: $28\frac{4}{5}$ Short division: 138 ÷ 6 = 23 

Examples of suggested formal written methods to be used by end of Key Stage 2

Taken from the appendix of the National Curriculum 2014.
Please note that this not an exhaustive list.

Addition and subtraction

789 + 642 becomes

$$\begin{array}{r} 7\ 8\ 9 \\ +\ 6\ 4\ 2 \\ \hline 1\ 4\ 3\ 1 \\ \hline 1\ 1 \end{array}$$

Answer: 1431

874 – 523 becomes

$$\begin{array}{r} 8\ 7\ 4 \\ -\ 5\ 2\ 3 \\ \hline 3\ 5\ 1 \end{array}$$

Answer: 351

932 – 457 becomes

$$\begin{array}{r} {\scriptstyle 8\ \ 12\ \ 1} \\ 9\ \not{3}\ \not{2} \\ -\ 4\ 5\ 7 \\ \hline 4\ 7\ 5 \end{array}$$

Answer: 475

932 – 457 becomes

$$\begin{array}{r} {\scriptstyle 1\ \ 1} \\ 9\ 3\ 2 \\ -\ \not{4}\ \not{5}\ 7 \\ {\scriptstyle 5\ \ 6} \\ \hline 4\ 7\ 5 \end{array}$$

Answer: 475

Short multiplication

24 × 6 becomes

$$\begin{array}{r} 2\ 4 \\ \times\ \ 6 \\ \hline 1\ 4\ 4 \\ \hline 2 \end{array}$$

Answer: 144

342 × 7 becomes

$$\begin{array}{r} 3\ 4\ 2 \\ \times\ \ 7 \\ \hline 2\ 3\ 9\ 4 \\ \hline 2\ \ 1 \end{array}$$

Answer: 2394

2741 × 6 becomes

$$\begin{array}{r} 2\ 7\ 4\ 1 \\ \times\ \ 6 \\ \hline 1\ 6\ 4\ 4\ 6 \\ \hline 4\ \ 2 \end{array}$$

Answer: 16 446

Long multiplication

24 × 16 becomes

$$\begin{array}{r} {\scriptstyle 2} \\ 2\ 4 \\ \times\ 1\ 6 \\ \hline 2\ 4\ 0 \\ 1\ 4\ 4 \\ \hline 3\ 8\ 4 \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ \ 2} \\ 1\ 2\ 4 \\ \times\ \ 2\ 6 \\ \hline 2\ 4\ 8\ 0 \\ 7\ 4\ 4 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ \ 1 \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ \ 2} \\ 1\ 2\ 4 \\ \times\ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ \ 1 \end{array}$$

Answer: 3224

Long division

432 ÷ 15 becomes

$$\begin{array}{r} 2\ 8\ \text{r}\ 12 \\ 15\overline{)\ 4\ 3\ 2} \\ 3\ 0\ 0 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \\ \hline 1\ 2 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r} 2\ 8 \\ 15\overline{)\ 4\ 3\ 2} \\ 3\ 0\ 0 \quad 15\times20 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \quad 15\times8 \\ \hline 1\ 2 \end{array}$$

$\frac{\not{12}}{\not{15}} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r} 2\ 8\cdot 8 \\ 15\overline{)\ 4\ 3\ 2\cdot 0} \\ 3\ 0\ \downarrow \\ \hline 1\ 3\ 2\ \downarrow \\ 1\ 2\ 0 \\ \hline 1\ 2\ 0 \\ \hline 0 \end{array}$$

Answer: 28·8

The sample Key Stage 2 mathematics test mark scheme (September 2015) suggests that children who get the answer correct will gain maximum points. If children get the answer wrong but show a suggested written method with an arithmetical error, they will gain one point. E.g.

Award **TWO** marks for the correct answer of 232

If the answer is incorrect, award **ONE** mark for the formal methods of division which contains no more than **ONE** arithmetical error, e.g.

- long division algorithm

wrong answer

$$\begin{array}{r} 13\overline{)\ 3016} \\ 26 \\ \hline 41 \\ -\ 39 \\ \hline 26 \\ -\ 26 \\ \hline 0 \end{array}$$

- short division algorithm

wrong answer

$$13\overline{)\ 3\ 0^{4}1^{2}6}$$